

Question Paper

Physics Multiple Correct (Maximum Marks: 32)

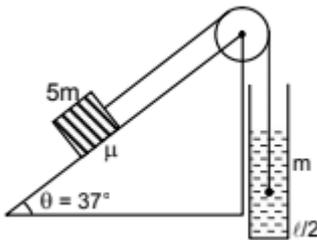
Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A block of mass 5 m is connected with a small solid spherical ball of mass m (density $=\rho$) by an ideal string passing over a smooth pulley. The block is on an inclined plane with coefficient of friction $\mu=0.02$. The spherical ball is in a liquid of density $\frac{\rho}{2}$. The coefficient of viscosity is η .

Block is released when string is just taut, then after some time it moves with constant velocity. Assume that the ball always remain inside the fluid. The radius of ball is r . Choose the correct option(s):



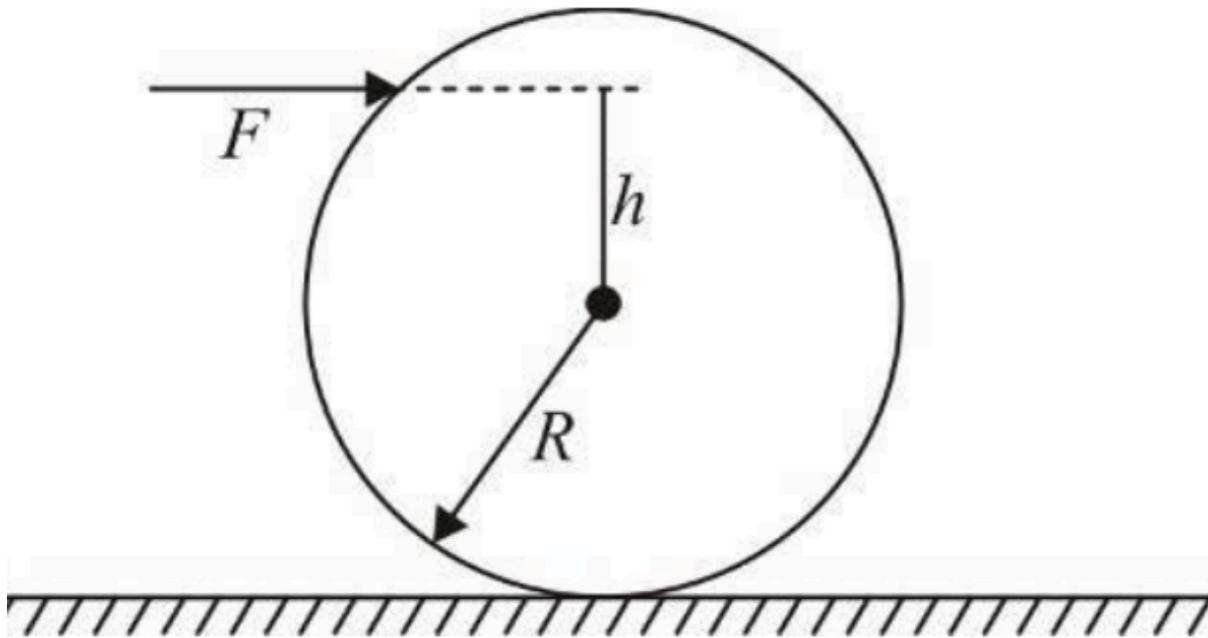
- A. Acceleration of the ball just after the release $\frac{17 \text{ m g}}{60}$
- B. Tension in string just after the release 2.2 m g
Tension in the string when ball move with constant velocity is 2.2 m g
- C. m g
- D. The constant velocity of the ball is $\frac{17 \text{ m g}}{60 \pi \eta r}$

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A solid sphere of mass M and radius R is placed on a rough horizontal surface. A horizontal force F is applied to the sphere at a height h ($0 \leq h \leq R$) from the centre as shown in figure. If the sphere rolls without slipping, then choose correct options.



- A. The maximum possible acceleration of the sphere is F / M
- B. The maximum possible acceleration of the sphere is $10 F / 7 M$
- The magnitude of maximum possible friction force acting on the sphere is
- C. $3 F / 7$
- The magnitude of maximum possible friction force action on the sphere is
- D. equal to F .

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The polonium isotope $({}_{84}^{210}\text{Po})$ is unstable. It emits an (α) -particle with energy of (5.30 MeV) . The atomic mass of $({}_{84}^{210}\text{Po})$ is (209.9829 u) and atomic mass of $({}_{2}^4\text{He})$ is (4.0026 u) , then choose the correct options.

- A. Daughter nucleus is $({}_{82}^{206}\text{Pb})$
- B. Disintegration energy is (5.30 MeV)
- C. Disintegration energy is (5.40 MeV)
- D. Mass of daughter nucleus is (205.9745 u)

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A spherical planet of radius (R) has spherically symmetrical distribution of mass density, varying as square of the distance from the centre, from zero at centre to maximum value (ρ_0) at its surface.

- The value of escape velocity of a mass (m) at the surface of planet is \
- A. $(\sqrt{\frac{4 \pi G \rho_0 R^2}{5}})$.
- The value of acceleration due to gravity (g) ' varies inside the
- B. planet as cube of the distance from centre.
- The value of escape velocity from surface is same as the escape velocity from another planet of same total mass (M) radius but having uniform
- C. mass density.
- The energy required to impart escape velocity to particles of masses (m) ' & $(2m)$ ', at the surface of planet, will be in ratio $(1: 2)$.
- D. & $(2m)$ ' ,

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

To study a relation of frequency and length of a given wire under constant tension using a sonometer, let you are performing an experiment in your school lab.

Now, choose the correct options.

- A. Tuning fork of known frequency is touched to wire for observing resonance
- B. Vibrations of wire are observed by paper riders
- Frequency of vibration of wire is $(f = \frac{1}{L D} \sqrt{\left(\frac{T}{\pi \rho}\right)})$ (where, $(L =)$ length of wire between bridges, $(D =)$
- C. diameter and $(\rho =)$ density of wire)
- D. Frequency of tuning forks remains constant with temperature of lab

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A cylindrical shaped wooden having mass (m) is floating inside a liquid with its axis is parallel to the vertical. The mass is depressed a little by a force and then force is removed, then (density of liquid is ρ , radius of cylinder is r)

- mass will execute SHM with time period inversely proportional to radius of
- A. cylinder
- B. mass will execute SHM with time period $(\frac{\pi r^2 \sqrt{g}}{2 m})$
- C. mass will not execute SHM
- mass will execute SHM with frequency $(\frac{1}{2 \pi} \sqrt{\frac{\pi r^2}{g \rho m}})$
- D. $(\frac{1}{2 \pi} \sqrt{\frac{\pi r^2}{g \rho m}})$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Pick the correct statements(s):

- The rms translational speed for all ideal-gases molecules at the same
- A. temperature is not the same but it depends on the mass
 - Each particle in a gas has average translational kinetic energy and the equation $\frac{1}{2} m v_{rms}^2 = \frac{3}{2} k T$ establishes the relationship between the translational kinetic energy of a particle and
 - B. temperature of an ideal gas
 - Temperature of an ideal gas is doubled from (100°C) to (200°C) . The average kinetic energy of each particle is
 - C. also doubled
 - It is possible for both the pressure and volume of a monoatomic ideal gas to change simultaneously without causing the internal energy of the gas to
 - D. change

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Two particles with mass (m_1) and (m_2) are connected by a mass less rigid rod of length (L) and placed on a horizontal frictionless table. At time $(t=0)$, the first mass receives an impulse perpendicular to the rod, giving it speed v . At this moment, the second mass is at rest. The minimum time after which the second mass will again come to rest is

- A. $t = \frac{2 \pi m_1 L}{(m_1 + m_2) v}$
- B. $t = \frac{2 \pi m_1 m_2 L}{(m_1 + m_2)^2 v}$
- C. $t = \frac{\pi (m_1 + m_2) L}{m_2 v}$
- D. $t = \frac{2 \pi L}{v}$

Physics Paragraph Type (Maximum Marks: 12)

Question No. 1

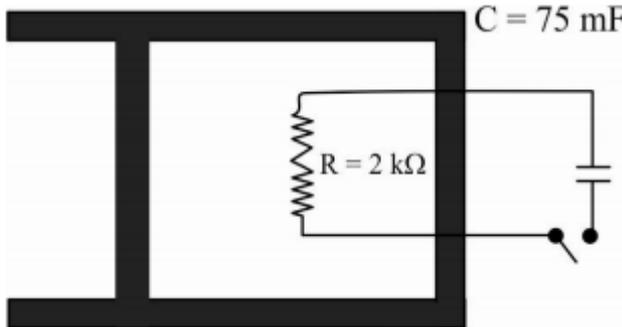
Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

A cylinder fitted with a piston which can slide without friction contains one mole of an ideal gas. The walls of the cylinder and piston are adiabatic. The cylinder contains a resistor of resistance $R = 2 \text{ k}\Omega$ which is connected to a capacitor of capacity $C = 75 \text{ mF}$. Initially, potential difference across capacitor is $\left(\frac{640}{3}\right) \text{ V}$ and switch is opened.

When switch is closed for $(2.5 \ln 4) \text{ min}$, then gas expands isobarically and its temperature increases by 72 K . Heat loss through the wires is negligible $\left(\frac{R}{8.3 \text{ J mol}^{-1} \text{ K}^{-1}}\right)$.



Question:

Work done by the gas is approximately :

- A. 0.2 kJ
- B. 0.4 kJ
- C. 0.6 kJ
- D. 0.8 kJ

Question No. 2

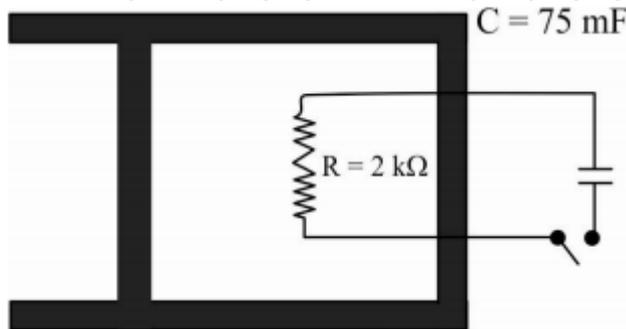
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Question:

The increment in internal energy of gas is :

- A. 1 kJ
- B. 2 kJ
- C. 3 kJ
- D. 4 kJ

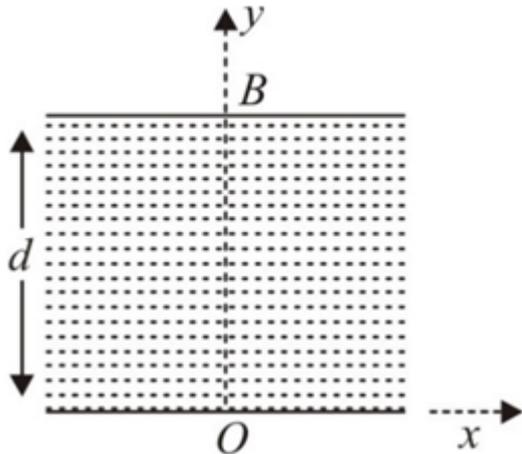
Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

The magnitude of velocity of water current in a river of width d varies across the river according to the relation $v = u_0 y / d$ ($0 \leq y \leq d$), where u_0 is the velocity of man in still water. The direction of water current is along x -axis.



Question:

The time taken by the man to reach from point O to B

O (travelling along $+y$ -axis) is :

- A. d / u_0
- B. $d / 2 u_0$
- C. $\pi d / 2 u_0$
- D. $\pi d / u_0$

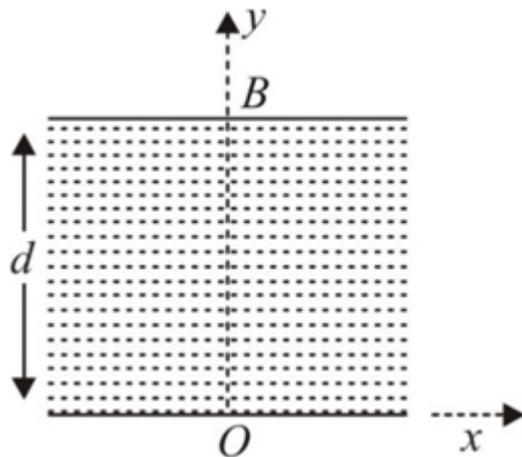
Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

The magnitude of velocity of water current in a river of width d varies across the river according to the relation $v = u_0 y / d$ ($0 \leq y \leq d$), where u_0 is the velocity of man in still water. The direction of water current is along x -axis.



Question:

The minimum time taken by the man to cross the river is :

- A. d / u_0
- B. $d / 2 u_0$
- C. $\pi d / 2 u_0$
- D. $\pi d / u_0$

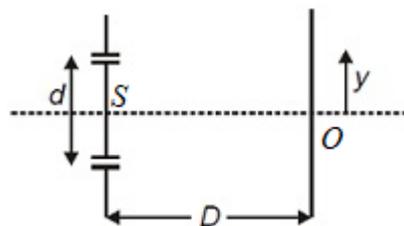
Physics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

In YDSE, if incident light consists of two wavelengths $\lambda_1 = 4000 \text{ \AA}$ and $\lambda_2 = 5600 \text{ \AA}$ and is parallel to line SO . The minimum distance y upon screen, measured from point O , will be where the bright fringe due to two wavelengths coincide is $\frac{n\lambda_1 D}{d}$. Find n .

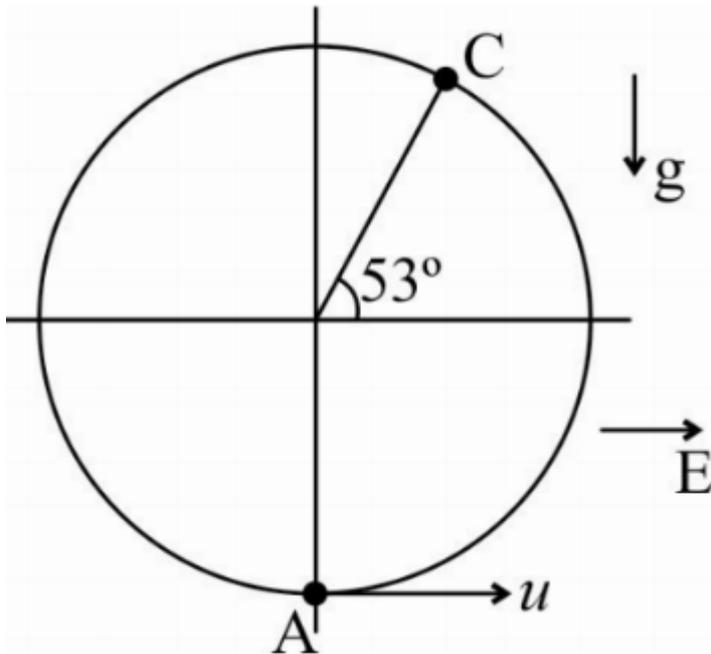


Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

A simple pendulum with a bob of mass $(m=2 \text{ kg})$, charge $(q=5 \text{ } \mu\text{C})$ and string of length $(l=1 \text{ m})$ is given a horizontal velocity (u) in uniform electric field $(E=2 \times 10^6 \text{ V/m})$ at its bottom most point A, as shown in figure. If particle leaves the circle at point C. Find $(\frac{u^2}{5} \cdot (g=10 \text{ m/s}^2))$



Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

Two radio signal broadcast their program at the same amplitude A and at slightly different frequency ω_1 , ω_2 respectively given that $\omega_2 - \omega_1 = \pi \times 10^2$. A detector detects the signal from the two stations simultaneously. It can only detect signal of intensity $I \geq 3A^2$. Detector remains idle for time $t_0 = \frac{\alpha}{\beta} \times 10^{-2} \text{ sec}$ in each cycle of the intensity of the signal. Here α and β are integers. Find minimum value of $\alpha + \beta$. (Use Intensity $I = A^2$) where I is intensity of each signal.

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

A Physical quantity P is given as - $P = \frac{x^2 y}{(4-t)^3}$ where, $x = (2 \pm 0.01) \text{ SI unit}$; $y = (4 \pm 0.02) \text{ SI unit}$; $t = (2 \pm 0.01) \text{ SI unit}$. Find the percentage error in calculation of P.

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

An isolated and charged spherical soap bubble has a radius ' r ' and the pressure inside is atmospheric. If ' T ' is the surface tension of soap solution, then charge on drop is $N \pi r \sqrt{2 r T \epsilon_0}$, then find the value of N .

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Consider a power line between two poles, the distance between the poles is L the linear density of the wire is μ , and the middle point of the wire hangs below the horizontal level at which the wire is fixed to the poles by a distance d . Assume that $d \ll L$. The tension T at the mid point of the wire is $\frac{L^2 \mu g}{n d}$. Find n .

Chemistry Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In which of the following pair(s), the minerals are converted into metals by self reduction?

- A. Ag_2S , ZnS
- B. Cu_2S , HgS
- C. PbS , ZnS
- D. Cu_2S , PbS

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

In how many of these reactions, a blue colouration/precipitate or compound is formed?

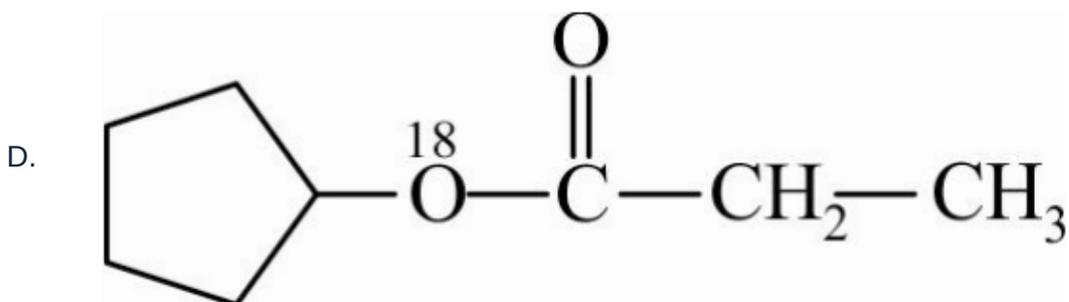
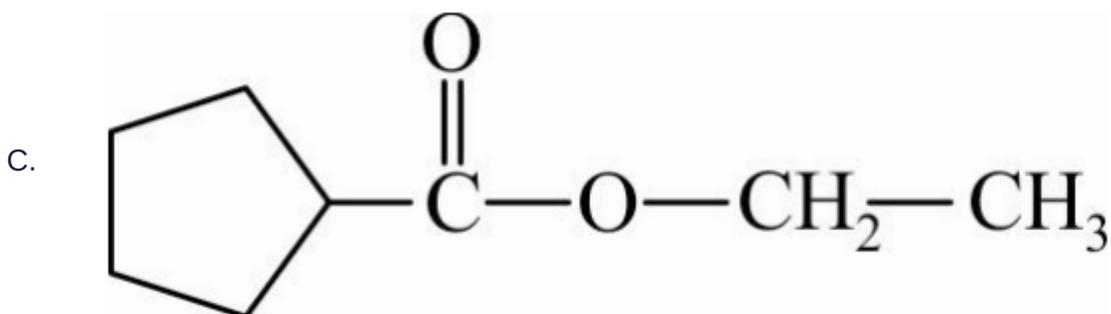
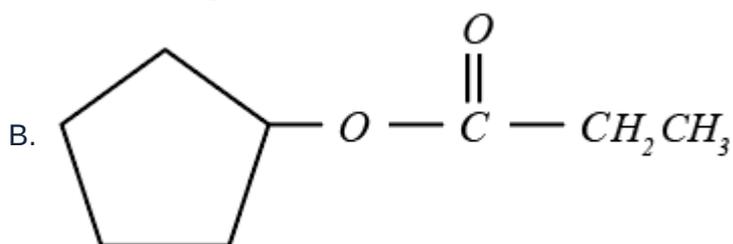
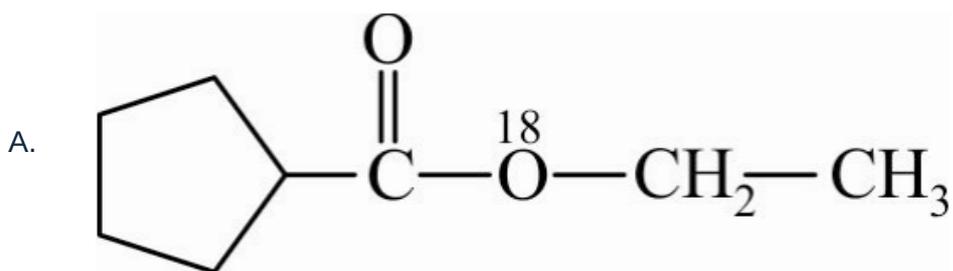
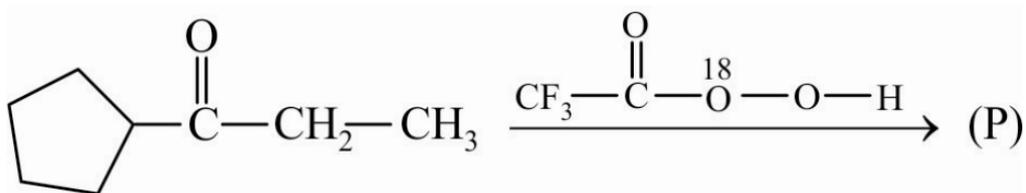
- A. Na (excess) + liq. $\text{NH}_3 \rightarrow \text{CoSO}_4 + \text{Na}_2\text{B}_4$
- B. $\text{O}_7 \xrightarrow{\Delta} \text{FeSO}_4 + \text{K}_4[\text{Fe}]$
- C. $(\text{CN})_6 \rightarrow$
- D. $\text{CuSO}_4 + \text{NH}_3$ (aq.) \rightarrow

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The major product (P) obtained in the following reaction is :



Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A flask containing some He gas at 1.5 atm and 300 K is connected to another flask, four times larger in volume containing nitrogen gas at same temperature and pressure by means of a narrow tube of negligible volume. The bigger flask was then kept in a thermostat bath maintained at 500 K , while other flask was maintained at constant 300 K .

Which of the following is/are correct regarding the final steady state conditions :

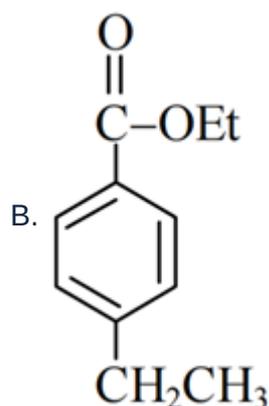
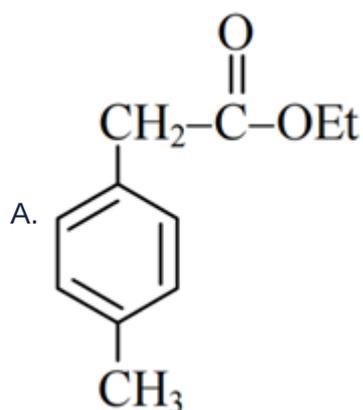
- A. Final pressure is 2.2 atm
- B. Ratio of moles of gases in smaller flask to bigger flask is $5: 12$
- C. Ratio of average KE of gas particles in smaller to bigger flask is $3: 5$
Total KE of gas particles in smaller flask is less than total KE of gas particles in bigger flask
- D. particles in bigger flask

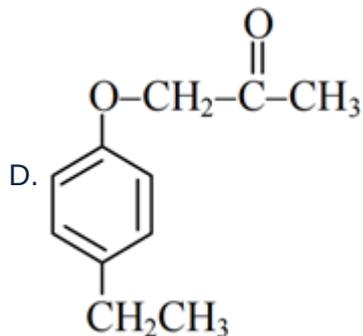
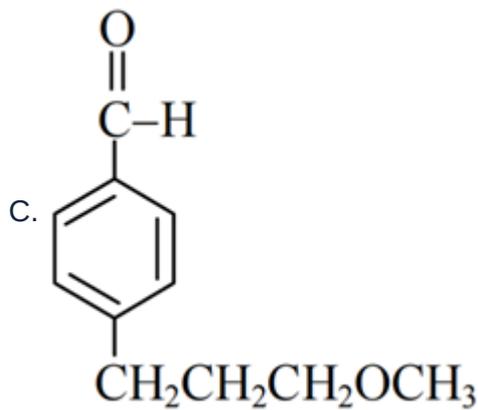
Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A compound (M.F. $\text{C}_{11}\text{H}_{14}\text{O}_2$) on hydrolysis gives a product, which on oxidation with acidified KMnO_4 gives terephthalic acid. The compound could be :





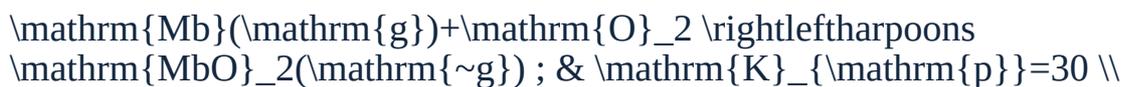
Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Myoglobin binds with oxygen and $\text{CO}(\text{g})$ present in air as

$\begin{array}{l} \end{array}$



\end{array}

When myoglobin at 1.0 atm is treated with air, 25% of it is bound to $\text{CO}(\text{g})$. If partial pressure of $\text{O}_2(\text{g})$ in air at equilibrium is 0.2 atm , then which of the following is/are true at equilibrium ?

- A. $P_{\text{MbO}_2} = 0.64 \text{ atm}$
- B. $P_{\text{CO}} = 2.5 \times 10^{-4} \text{ atm}$
- C. $P_{\text{Mb}} = 0.11 \text{ atm}$
- D. $P_{\text{T}} = 1.32 \text{ atm}$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Nitrogen can be obtained by heating :

- A. $\text{Pb} \left(\text{N}_3 \right)_2$
- B. KNO_3
 $\left(\text{NH}_4 \right)_2 \text{Cr}_2 \text{O}_7$
- C. O_7
- D. $\text{NH}_4 \text{NO}_2$

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Let the color of the indicator HIn (colorless) will be visible only when its ionized form (pink) is 25% or more in a solution. Suppose $\text{HIn} \left(\text{pK}_a = 9.0 \right)$ is added to a solution of $\text{pH} = 9.4$ predict what will happen. (Take $\log 2 = 0.3$)

- A. Pink color will be visible
- B. Pink color will not be visible
- C. % of ionized form will be less than 25%
- D. % of ionized form will be more than 25%

Chemistry Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Two inorganic compounds (A) and (B) are obtained by treating calcium phosphate with magnesium metal. (A) on hydrolysis produces a gas (C). The gas (C) was burnt in excess air to yield compound (D). (D) reacts with (B) to produce (E). [Atomic mass of $\text{P} = 31 \text{ g}$, $\text{Ca} = 40 \text{ g}$, $\text{Mg} = 24 \text{ g}$, $\text{O} = 16 \text{ g}$]

Question:

The gas C reacts with $\text{CuSO}_4(\text{aq})$ solution to give a black precipitate of compound X . hence, X is:

- A. $\text{Cu}_3 \text{P}$
- B. $\text{Cu}_2 \text{P}_3$
- C. $\text{Cu}_3 \text{P}_2$
- D. $\left[\text{Cu} \left(\text{PH}_3 \right)_4 \right] \text{SO}_4$

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Two inorganic compounds (A) and (B) are obtained by treating calcium phosphate with magnesium metal. (A) on hydrolysis produces a gas (C). The gas (C) was burnt in excess air to yield compound (D). (D) reacts with (B) to produce (E). [Atomic mass of $\mathrm{P}=31 \text{ g}$, $\mathrm{Ca}=40 \text{ g}$, $\mathrm{Mg}=24 \text{ g}$, $\mathrm{O}=16 \text{ g}$]

Question:

If 38.4 g of magnesium is taken initially, then the maximum weight of compound (E) formed (assuming 100 per cent efficiency of all the reactions) is :

- A. 48.2 g
- B. 36.4 g
- C. 39.5 g
- D. 14 g

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

For given solution in the questions molarity can be treated as molality and solute(s) is/are nonvolatile in nature.

Given that : K_a of $\text{CH}_3\text{COOH} = 10^{-5}$; $K_f =$ Molal depression constant K_a of $\text{HCOOH} = 10^{-4}$; $K_b =$ Molal elevation constant

Column - I	Column - II	Column - III
(I) $\Delta T_b \approx 0.3$	(A) Metal becomes deposited	(I) Colligative properties
(II) $\pi \approx 0.4 \text{ RT}$	(P) $\text{pH} = 8.5$ at cathode	(II) One of the product formed during electrolysis at Pt - electrode
(III) $\Delta T_f \approx 0.2$	(Q) $\text{pH} = 4.7$	(III) Nature of aqueous solution at 25 C
(IV) $\pi \approx 0.3 \text{ RT}$	(R) $\text{pH} \approx 7$	(IV) H_2 gas at cathode
(V) $\Delta T_b \approx 0.2$	(S) Weakly basic solution	(V) O_2 gas at anode

Which of the following option is correctly matched for 0.1 M HCOOK (aq.)

- A. (I), (B), (S)
- B. (IV), (B), (S)
- C. (III), (D), (Q)
- D. (III), (C), (P)

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

For given solution in the questions molarity can be treated as molality and solute(s) is/are nonvolatile in nature.

Given that : K_a of $\text{CH}_3\text{COOH} = 10^{-5}$; $K_f =$ Molal depression constant

K_a of $\text{HCOOH} = 10^{-4}$; $K_b =$ Molal elevation constant

Column - I	Column - II	Column - III
(I) $\Delta T_b \approx 0.3 K_b$ & (A) Metal becomes deposited at cathode	(B) $\text{pH} = 8.5$	(P) $\text{pH} = 8.5$
(II) $\pi \approx 0.4 RT$ & (B) H_2 gas at cathode	(Q) $\text{pH} = 4.7$	(R) $\text{pH} \approx 7$
(III) $\Delta T_f \approx 0.2 K_f$ & (C) H_2 gas at anode	(S) Weakly basic solution	(D) O_2 gas at anode

Select the correct match for aq. solution having $0.1 M \text{CH}_3\text{COONa}$ and $0.2 M \text{CH}_3\text{COOH}$.

- A. (II), (B), (Q)
- B. (II), (A), (Q)
- C. (IV), (A), (S)
- D. (I), (C), (R)

Chemistry Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

Consider the following species NO_3^- , AsO_3^{3-} , CO_3^{2-} , ClO_3^- , SO_3^{2-} and BO_3^{3-}

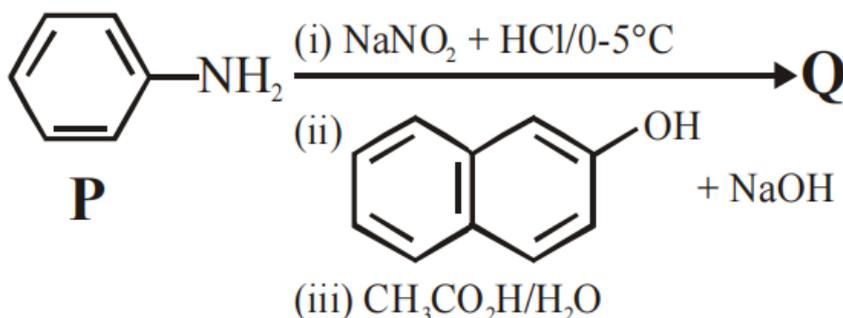
How many of the following are non planar?

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Consider the reaction sequence from \mathbf{P} to \mathbf{Q} shown below. The overall yield of the major product \mathbf{Q} from \mathbf{P} is 75%. What is the amount in grams of \mathbf{Q} obtained from 9.3 mL of \mathbf{P} ? (Use density of \mathbf{P} = 1.00 g mL⁻¹, Molar mass of \mathbf{C} = 12.0, \mathbf{H} = 1.0, \mathbf{O} = 16.0 and \mathbf{N} = 14.0 g mol⁻¹)



Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

The unit cell of hydrated $\text{TlAl}(\text{SO}_4)_2$ is face centered with unit cell length 'a' = 1.19 nm. If the density of the salt is 2.32 g/cc, then the approximate number of water of hydration units per $\text{TlAl}(\text{SO}_4)_2$ unit will be (Molar mass of $\text{TlAl}(\text{SO}_4)_2$ is 423.5). And convert the answer to nearest integer.

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

At 298 K, if ΔG_f° of $\text{HCl}(g)$ is 1.72 kJ mol⁻¹, then calculate K_p for the following reversible reaction: $2 \text{HCl}(g) \rightleftharpoons \text{H}_2(g) + \text{Cl}_2(g)$

(Use: at 298 K: $2.303 R T = 5700 \text{ J mol}^{-1}$ and $\log 2 = 0.30$)

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

An optically active substance 'A' is decomposing into optically active substance 'B' and 'C' as



The specific rotations of A , B and C are $+40^\circ$, $+10^\circ$ and -30° per mole, respectively. Initially 'A' and 'C' were present in 4: 3 mole ratio. Close to how many hours the sample becomes optically inactive ?

[Given : $\left. \operatorname{ell}_D^{20} \frac{7}{5} = 0.34, \operatorname{ell}_D^{20} \frac{20}{13} = 0.42 \right]$

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

Chlorobutane undergoes chlorination in presence of light. Total number of possible dichloro products are:

Mathematics Multiple Correct (Maximum Marks: 32)

Question No. 1

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

Area bounded by the circle which is concentric with the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and which passes through $\left(4, -\frac{9}{5}\right)$, the vertical chord common to both circle and ellipse on the positive side of x -axis is

- A. $\frac{481}{25} \tan^{-1} \left(\frac{9}{20} \right) - \frac{36}{5}$
- B. $2 \tan^{-1} \left(\frac{9}{20} \right)$
- C. $\frac{481}{25} \tan^{-1} \left(\frac{9}{20} \right)$
- D. none of these

Question No. 2

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If E and F are two independent events; such that

$$P(E \cap F) = \frac{1}{6}, P(E^c \cap F^c) = \frac{1}{3}$$

and $(P(E) - P(F))(1 - P(F)) > 0$, then

- A. $P(E) = \frac{1}{2}$
- B. $P(E) = \frac{1}{4}$
- C. $P(F) = \frac{1}{3}$
- D. $P(F) = \frac{2}{3}$

Question No. 3

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

$I_k = \int_{-k\pi}^{k\pi} \sin x |\sin x| dx$, where $[.]$ is greatest integer function and $I_n = \sum_{k=1}^n I_k = a - b((n+c)!)^2$, then

- A. value of $a+b+c$ is 7
- B. $\lim_{n \rightarrow \infty} \frac{I_n}{(n+1)!}$ is -2
- C. $I_n < 0, \forall n \in \mathbb{N}$
- D. $\int e^k \frac{I_k}{I_{k+1}} dk$ is equal to $\frac{e^k}{k+1} + A$,
(A is integration constant)

Question No. 4

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A curve for which the intercept cut off by any tangent on y -axis is proportional to the square of the ordinate of the point of tangency, then curve is

- A. $\frac{y}{x} = kx + c$
- B. $\frac{y}{x} = kx + c$
- C. $\frac{C_1}{x} - \frac{C_2}{y} = x$
- D. $\frac{c_1}{x} + \frac{c_2}{y} = 1$

Question No. 5

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

The images of focus of parabola $y^2 = 4ax$ in the tangents drawn at extremities of a focal chord are joined to form a line segment PQ , then

- A. $PQ \geq$ latus rectum
- B. PQ lies on directrix

- C. PQ subtends right angle at focus
 D. $\angle POQ \geq \tan^{-1}\left(\frac{4}{3}\right)$

Question No. 6

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

A function $f(x)$ is defined by $f(x) = \int_0^{\pi} \cos t \cos(x-t) dt$, $0 \leq x \leq 2\pi$ then which of the following hold(s) good?

- A. $f(x)$ is continuous and differentiable in $(0 < x < 2\pi)$
 B. Range of $f(x)$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
 C. Number of solutions of $f(x) = \cos x$ in $(x \in (0, 2\pi))$ is 2
 $f(x) = \frac{\pi}{2} \cos x$ holds good in $(x \in \left(0, \frac{\pi}{2}\right))$
 D. $\cup \left(\frac{3\pi}{2}, 2\pi\right)$

Question No. 7

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If $f: \mathbb{R} \rightarrow [a, 6]$ defined by $f(x) = \frac{x^2 - 2x + d}{x^2 + 3x + d}$ is an onto function, then

- A. a is equal to $\frac{2}{7}$
 B. d is equal to 4
 C. a is equal to 1
 D. $(d - 7a)$ is equal to -3

Question No. 8

One or More Options Correct Type

The question has multiple options out of which ONE or MORE is/are correct.

If a, b, c, d are four unequal positive numbers which are in A.P. then

- A. $\frac{1}{a} + \frac{1}{d} = \frac{1}{b} + \frac{1}{c}$
 B. $\frac{1}{a} + \frac{1}{d} < \frac{1}{b} + \frac{1}{c}$
 C. $\frac{1}{a} + \frac{1}{d} > \frac{1}{b} + \frac{1}{c}$
 D. $\frac{1}{b} + \frac{1}{c} > \frac{4}{a+d}$

Mathematics Paragraph Type (Maximum Marks: 12)

Question No. 1

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Let a point P whose position vector is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is called lattice point if $x, y, z \in \mathbb{N}$. If atleast two of x, y, z are equal then this lattice point is called isosceles lattice point. If x, y, z are equal then this lattice point is called equilateral lattice point.

Question:

Which of following is (are) correct?

- The number of lattice point on the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 10$ are 45
- The number of lattice point on the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 10$ are 36
- Area of triangle formed by isosceles lattice points lying on the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$ is $\frac{\sqrt{3}}{2}$
- Option 2 and 3

Question No. 2

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

Let a point P whose position vector is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is called lattice point if $x, y, z \in \mathbb{N}$. If atleast two of x, y, z are equal then this lattice point is called isosceles lattice point. If x, y, z are equal then this lattice point is called equilateral lattice point.

Question:

If a lattice point is selected at random from lattice points which satisfy $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) \leq n$, then the probability that the selected lattice point is equilateral given that it is isosceles lattice point is :

- A. $\frac{1}{21}$ is $n=11$
- B. $\frac{1}{22}$ is $n=11$
- C. $\frac{1}{17}$ is $n=10$
- D. $\frac{1}{18}$ is $n=10$

Question No. 3

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

The n^{th} roots of unity are $\cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}$, $(k=0,1,2, \dots, n-1)$, so that for n odd

$$x^{n-1} = (x-1) \prod_{k=1}^{\frac{n-1}{2}} \left(x - \cos \frac{2k\pi}{n} - i \sin \frac{2k\pi}{n} \right) \left(x - \cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n} \right)$$

or $\quad x^{\frac{n}{2}} - x^{-\frac{n}{2}} = \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}} \right) \prod_{k=1}^{\frac{n-1}{2}} \left(x + \frac{1}{x} - 2 \cos \frac{2k\pi}{n} \right)$
and hence writing $x = \cos 2\theta + i \sin 2\theta$, we get

$$\sin n\theta = 2^{\frac{n-1}{2}} \sin \theta \prod_{k=1}^{\frac{n-1}{2}} \left(\cos 2\theta - \cos \frac{2k\pi}{n} \right), \quad (n \text{ odd})$$

Similarly for n even

$$\sin n\theta = 2^{\frac{n-2}{2}} \sin 2\theta \prod_{k=1}^{\frac{n-2}{2}} \left(\cos 2\theta - \cos \frac{2k\pi}{n} \right)$$

Question:

The value of $\sin \frac{\pi}{7} \sin \frac{2\pi}{7} \sin \frac{3\pi}{7}$ is equal to :

- A. $\frac{7}{8}$
- B. $\frac{\sqrt{7}}{4}$
- C. $\frac{\sqrt{7}}{8}$
- D. $\frac{7}{64}$

Question No. 4

Only One Option Correct Type

Each question has multiple options out of which ONLY ONE is correct.

Paragraph:

The n^{th} roots of unity are $\cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n}$, $(k=0,1,2, \dots, n-1)$, so that for n odd

$$x^n - 1 = (x-1) \prod_{k=1}^{n-1} \left(x - \cos \frac{2k\pi}{n} - i \sin \frac{2k\pi}{n} \right) \left(x - \cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n} \right)$$

or $x^{n/2} - x^{-n/2} = \left(x^{1/2} - x^{-1/2} \right) \prod_{k=1}^{n-1} \left(x + \frac{1}{x} - 2 \cos \frac{2k\pi}{n} \right)$
and hence writing $x = \cos 2\theta + i \sin 2\theta$, we get

$$\sin n\theta = 2^{\frac{n-1}{2}} \sin \theta \prod_{k=1}^{\frac{n-1}{2}} \left(\cos 2\theta - \cos \frac{2k\pi}{n} \right), \quad (n \text{ odd})$$

Similarly for n even

$$\sin n\theta = 2^{\frac{n-2}{2}} \sin 2\theta \prod_{k=1}^{\frac{n-2}{2}} \left(\cos 2\theta - \cos \frac{2k\pi}{n} \right)$$

Question:

If $\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} = k \cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{4\pi}{9}$, then k is equal to :

- A. -1
- B. 1
- C. 2
- D. -2

Mathematics Numerical (Maximum Marks: 24)

Question No. 1

Numerical Type

The answer has to be filled into the input box provided below.

If $f: \mathbb{R} - \{0,1\} \rightarrow \mathbb{R}$ be a function satisfying $f(x) + f\left(1 - \frac{1}{x}\right) = \tan^{-1} x$ and $f(N) = f\left(\frac{1}{2}\right)$, then the value of $f\left(\frac{8}{N}\right)$ is

Question No. 2

Numerical Type

The answer has to be filled into the input box provided below.

Let \mathbf{A} , \mathbf{B} and \mathbf{C} be 3 square matrices of order 3×3 . If $\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + \mathbf{B} = \mathbf{0}$ and $(\operatorname{adj}(\operatorname{adj}(\mathbf{B}))) = \mathbf{C}$, where $\mathbf{A} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 2 \\ 0 & 3 & 0 \end{bmatrix}$, then $\left(\frac{\text{sum of the elements of } \mathbf{C}}{10}\right)$ is

Question No. 3

Numerical Type

The answer has to be filled into the input box provided below.

If \mathbf{N} is the number of ways in which 5 distinct balls can be distributed among 3 people such that no person is left empty handed, then $\frac{\mathbf{N}}{10}$ is equal to

Question No. 4

Numerical Type

The answer has to be filled into the input box provided below.

Let $f(x) = (x-1)|\operatorname{sgn}(x)|$ and $g(x) = \max_{t \in \mathbb{R}, -\infty < t \leq x, -\infty < x < \infty} f(t)$, then the number of points where $g(x)$ is non-differentiable is

$\operatorname{sgn}(x)$ represents signum function of x

Question No. 5

Numerical Type

The answer has to be filled into the input box provided below.

Let $I = \int_{-3}^{-2} \left(\frac{x^2 - x}{x^3 - 3x + 1}\right)^2 \left(1 + \frac{1}{x^2} + \frac{1}{(1-x)^2}\right) dx$ then the greatest integer less than the value of I , is

Question No. 6

Numerical Type

The answer has to be filled into the input box provided below.

The sequence $\{a_n\}$ is defined by $a_{k+1} = a_k^2 + a_k$, $a_1 = \frac{1}{2}$ and $P = \left[\frac{1}{a_1+1} + \frac{1}{a_2+1} + \dots + \frac{1}{a_{100}+1}\right]$. (where $[.]$ the greatest integer function). The value of $P+4$ is